

Hong Kong Mathematics Olympiad (2001 – 2002)

Heat Event (Individual)

香港数学竞赛 (2001 – 2002)

初赛项目(个人)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 袋中有数字卡 9 张，其数字分别为 1 至 9。若随机一次抽出 3 张，求被抽出的卡的数字全是奇数的概率。（答案以分数表达，并化至最简。）

There are 9 cards, numbered from 1 to 9, in a bag. If 3 cards are drawn together at random, find the probability that all are odd. (Express your answer in the simplest fraction.)

2. 已知 $a^3 = 150b$ ，且 a 和 b 都是正整数。求 b 的最小值。

Given $a^3 = 150b$, and a, b are positive integers, find the least value of b .

3. 已知 $\cos 15^\circ = \frac{\sqrt{a} + \sqrt{b}}{4}$ ，且 a, b 是自然数。若 $a + b = y$ ，求 y 的值。

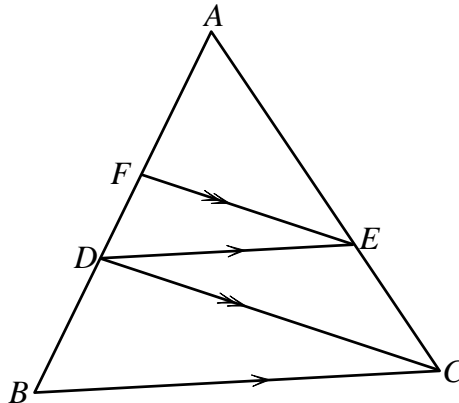
Suppose $\cos 15^\circ = \frac{\sqrt{a} + \sqrt{b}}{4}$, and a, b are natural numbers. If $a + b = y$, find the value of y .

4. 把数字 2, 3, 4, 5 组成没有重复数字的四位数，求这些四位数的和。

Each of the digits 2, 3, 4, 5 can be used once and only once in writing a four-digit number. Find the sum of all such numbers.

5. 在 $\triangle ABC$, $DE \parallel BC$, $FE \parallel DC$, $AF = 2$, $FD = 3$ 和 $DB = X$ 。求 X 的值。

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6. 若一圆内接四边形的四边长度为 9, 10, 10 和 21, 求该圆内接四边形的面积。

If the lengths of the sides of a cyclic quadrilateral are 9, 10, 10 and 21 respectively, find the area of the cyclic quadrilateral.

7. 若 $\frac{(a-b)(c-d)}{(b-c)(d-a)} = 3$, 求 $\frac{(a-c)(b-d)}{(a-b)(c-d)}$ 的值。

If $\frac{(a-b)(c-d)}{(b-c)(d-a)} = 3$, find the value of $\frac{(a-c)(b-d)}{(a-b)(c-d)}$.

8. 若 $x^3 + kx^2 + 3$ 除以 $x+3$, 其余数较被 $x+1$ 除所得的余数少 2。求 k 的值。

When the expression $x^3 + kx^2 + 3$ is divided by $x+3$, the remainder is 2 less than when divided by $(x+1)$. Find the value of k .

9. 已知圆形上的某扇形的周界为 18。当圆的半径为 r 时, 该扇形的面积达至最大值, 求 r 的值。

Given that the perimeter of a sector of a circle is 18. When the radius is r , the area of the sector attains the maximum value, find the value of r .

10. 已知 $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$, 求 $f(5)$ 的值。

Given $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$, find the value of $f(5)$.